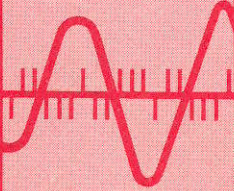


S. Bittanti · A. J. Laub  
J. C. Willems (Eds.)

# The Riccati Equation



Springer-Verlag

# Communications and Control Engineering Series

Editors: A. Fettweis · J. L. Massey · J.W. Modestino · M. Thoma





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*THE*  
*RICCATI EQUATION*

Springer-Verlag  
Berlin Heidelberg New York  
London Paris Tokyo  
Hong Kong Barcelona Budapest

*Editors:*

Sergio Bittanti  
Politecnico di Milano, Dipartimento di Elettronica,  
Piazza Leonardo da Vinci 32, I-20133 Milano, Italy

Alan J. Laub  
Department of Electrical and Computer Engineering,  
University of California, Santa Barbara, CA 93106, USA

Jan C. Willems  
Department of Mathematics, University of Groningen,  
P.O. Box 800, NL-9700 AV Groningen, The Netherlands

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Frontispiece: Portrait of Count Jacopo Riccati reproduced by  
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ISBN 3-540-53099-1 Springer-Verlag Berlin Heidelberg New York  
ISBN 0-387-53099-1 Springer-Verlag New York Berlin Heidelberg

Library of Congress Cataloging-in-Publication Data  
The Riccati equation / S. Bittanti, A. J. Laub, J. C. Willems, editors.  
(Communications and control engineering series)  
Includes index.  
ISBN 3-540-53099-1  
ISBN 0-387-53099-1  
I. Riccati equation. II. Bittanti, Sergio. III. Laub, Alan J.  
III. Willems, Jan C. IV. Series.  
1948-  
QA371.R36 1991

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Printed in Germany

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Typesetting: Text supplied by authors, formatted by Danny L. Lewis, Buchproduktion Berlin, with  $\text{T}_{\text{E}}\text{X}$ ;  
Printing: Color-Druck Dorfi GmbH, Berlin; Binding: Lüderitz & Bauer, Berlin  
61/3020-543210

## Preface

Conceived by Count Jacopo Francesco Riccati more than a quarter of a millennium ago, the Riccati equation has been studied in subsequent centuries by a number of distinguished scientists, including several members of the Bernoulli family, L. Euler and J. Liouville, to name but a few. The last thirty years have witnessed a considerable renaissance of the research around this equation, stemming from the widespread developments in and diffusion of systems and control concepts and applications. Indeed, since its introduction in control theory at the beginning of the sixties, the Riccati equation has known an impressive range of applications, such as linear quadratic optimal control, stability theory, stochastic filtering and stochastic control, synthesis of linear passive networks, differential games and, more recently,  $H_\infty$ -control and robust stabilization.

The state of the art in this area was reviewed at a recent Workshop on the Riccati Equation in Control, Systems, and Signals held in Como (Italy) June 26-28, 1989. This workshop constituted the foundation from which the idea of this book germinated.

The purpose of this book is to present a self-contained treatment of the main issues evolving around the Riccati equation, in particular theory, applications, and numerical algorithms. The book, which consists of coordinated tutorial chapters written by different authors, is intended as a graduate text as well as a reference for scientists, especially engineers, and mathematicians.

The organization is as follows. Chapter 1 is devoted to the history and pre-history of the Riccati equation. Chapters 2 and 3 supply a comprehensive view of the algebraic Riccati equation, mainly based on a linear algebra approach. A geometrical analysis of the equation is carried out in Chapters 4 and 5. Chapters 2 to 5 deal with the constant coefficient case. The periodically time-varying Riccati equation is the subject of Chapter 6. The leading numerical techniques for the solution of the Riccati equation are overviewed in Chapter 7. The remaining four chapters address connections between the Riccati equation and some important problems in systems and control. More precisely, in Chapter 8, the role of the Riccati equation in the study of dissipative systems is elucidated. The linear quadratic optimal control problem in its various facets is the subject of Chapters 9 and 10. Finally, a unified survey on generalized Riccati equations in dynamic games is presented in Chapter 11.

The book is reasonably self-contained, but the reader should have some familiarity with basic concepts in system theory and linear algebra. Section 2 of

## VI Preface

Chapter 2 presents a concise overview of some of these concepts. Some knowledge of geometric concepts is also useful, mainly for Chapters 4 and 5.

The editors would like to express their sincere thanks and acknowledgment to all authors for care and diligence in providing their contribution.

Sergio Bittanti, Alan J. Laub and Jan C. Willems

## List of Contributors

*Gregory Ammar*

Department of Mathematics, Northern Illinois University, DeKalb, IL 60115,  
USA

*Tamer Basar*

University of Illinois at Urbana-Champaign, Decision and Control Laboratory,  
1101 West Springfield Avenue, Urbana, IL 61801, USA

*Robert Bitmead*

Department of Systems Engineering, Australian National University,  
Canberra ACT 2601, Australia

*Sergio Bittanti*

Politecnico di Milano, Dipartimento di Elettronica, Piazza Leonardo da Vinci 32,  
20133 Milano, Italy

*Frank M. Callier*

Department of Mathematics, FNDP, Rempart de la Vierge 8,  
B-5000 Namur, Belgium

*Patrizio Colaneri*

Centro di Teoria dei Sistemi, Dipartimento di Elettronica,  
Piazza Leonardo da Vinci 32, 20133 Milano, Italy

*Giuseppe De Nicolao*

Centro di Teoria dei Sistemi, Dipartimento di Elettronica,  
Piazza Leonardo da Vinci 32, 20133 Milano, Italy

*Michel Gevers*

Université Catholique de Louvain, Bâtiment Maxwell,  
B-1348 Louvain-la-Neuve, Belgium

*Vladimir Kucera*

Institute of Information Theory and Automation, Czechoslovak Academy of  
Sciences, P.O. Box 18, vodárenskou věží 4, 18208 Prague, Czechoslovakia

*Peter Lancaster*

University of Calgary, Department of Mathematics and Statistics,  
2500 University Drive NW, Calgary, Alberta, Canada T2N 1N4



X List of Contributors

*Alan J. Laub*

Department of Electrical and Computer Engineering, University of California,  
Santa Barbara CA 93106, USA

*Clyde F. Martin*

Department of Mathematics, Texas Tech University, Box U319,  
Lubbock, Texas 79409, USA

*Leiba Rodman*

Department of Mathematics, College of William and Mary,  
Williamsburg, Virginia, 23185, USA

*Mark A. Shayman*

Department of Electrical Engineering, University of Maryland,  
College Park MD 20724, USA

*H. L. Trentelman*

Mathematics Institute, P.O. Box 800,  
NL-9700 AV Groningen, The Netherlands

*Jacques L. Willems*

Engineering Faculty, University of Gent, Grotesteenweg-Noord 2,  
B-9710 Gent, Belgium

*Jan C. Willems*

University of Groningen, Department of Mathematics, P.O. Box 800,  
NL-9700 AV Groningen, The Netherlands

Conceived by Count Jacopo Francesco Riccati more than a quarter of a millennium ago, the Riccati equation has been widely studied in subsequent centuries. Since its introduction in control theory in the sixties, the matrix Riccati equation has known an impressive range of applications, such as optimal control,  $H_\infty$  optimization and robust stabilization, stochastic realization, and synthesis of linear passive networks, to name but a few.

This book consists of 11 chapters surveying the main concepts and results related to the matrix Riccati equation, both in continuous and discrete time. Theory, applications, and numerical algorithms are extensively presented in an expository way. As a foreword, the history and prehistory of the Riccati equation are concisely presented.

ISBN 3-540-53099-1

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