

Lecture Notes in Control and Information Sciences

Edited by M. Thoma and A. Wyner

86

Time Series and Linear Systems

Edited by Sergio Bittanti



Springer-Verlag

copie con correzioni

Lecture Notes in Control and Information Sciences

Edited by M. Thoma and A. Wyner

86

Time Series and
Linear Systems

*Sergio Bittanti
Milano*

Edited by S. Bittanti



Springer-Verlag
Berlin Heidelberg New York
London Paris Tokyo

Series Editors

M. Thoma · A. Wyner

Advisory Board

L. D. Davisson · A. G. J. MacFarlane · H. Kwakernaak
J. L. Massey · Ya Z. Tsytkin · A. J. Viterbi

Editor

Sergio Bittanti
Dipartimento di Elettronica
Politecnico di Milano
Piazzo Leonardo da Vinci 32
20133 Milano (Italy)

ISBN 3-540-16903-2 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-16903-2 Springer-Verlag New York Berlin Heidelberg

Library of Congress Cataloging in Publication Data

Time series and linear systems.
(Lecture notes in control and information sciences; 86)
Includes bibliographies.

1. Time-series analysis. 2. Linear systems.

I. Bittanti, Sergio. II. Series.

QA280.T558 1986 519.5'5 86-20244

ISBN 0-387-16903-2 (U.S.)

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically those of translation, reprinting, re-use of illustrations, broadcasting, reproduction by photocopying machine or similar means, and storage in data banks. Under § 54 of the German Copyright Law where copies are made for other than private use, a fee is payable to "Verwertungsgesellschaft Wort", Munich.

© Springer-Verlag Berlin, Heidelberg 1986

Printed in Germany

Offsetprinting: Color-Druck, G. Baucke, Berlin

Binding: B. Helm, Berlin

2161/3020-543210

PREFACE

Over the five past years, a stream of research at the Politecnico di Milano (Italy) has been in the methodology of modelling and identification of time series by means of linear systems. Several specialists of different backgrounds, including System and Control Theory, Statistics, Econometrics, Numerical Analysis, visited the Politecnico contributing with their talks to setting up a workshop on the subject. The train of ideas underlying this activity was to develop a system-theoretic point of view for the art of modelling.

This book is a partial report of such an activity. The various chapters are extended introductory papers overviewing important advanced topics in the field. They also constitute useful introductions to research directions of current interest.

The book is organized as follows. The first chapter is an introduction to the use of stochastic models in time series analysis. The problem of modelling is interpreted here as the problem of finding the linear model which is the best approximant for the data at hand. Among other things the use of criteria such as AIC or BIC is critically discussed. Moreover, the problem of determining a suitable rational transfer function approximation is studied as the problem of approximating the infinite Hankel matrix of the impulse response coefficients with a Hankel matrix of finite rank. Linear systems where all observed variables are subject to errors are considered in the second chapter. The motivation is that prejudicial causality assumptions can then be avoided. A new class of dynamic models

for time series is proposed in the third chapter. These models are based on the classical Factor Analysis approach, and are strictly related to the systems introduced in Chapter 2. The fourth chapter is devoted to the so called Minimum Description Length approach. A model is then judged by the number of binary digits with which it permits to encode the observed data. This leads to the notion of stochastic complexity of the data, as the shortest number of binary digits with which it permits to encode the observed data. Chapter 5 deals with systems with periodically time-varying coefficients, which can be used to describe seasonal time series. The attention focuses here on the basic structural properties of these systems, i.e. reachability, stabilizability and so on. The role played by these properties in the analysis of stochastic periodic systems is touched upon. Some numerical problems in linear system theory are considered in the sixth chapter. An extensive overview of the LU, QR, Shur and Singular Value Decomposition algorithms is provided. Then, the problem of computing the reachability subspace of a time-invariant system is studied. The last chapter is devoted to the discussion of some recent trends in Econometrics.

The volume can be used either as a textbook for monographic courses on the subject or as a reference book providing researchers with the main trends and perspectives in the field.

The editor expresses his sincere acknowledgment to the fellow authors for their most valuable contributions, as well as their care and patience in the preparation of the manuscripts.

The support of the Centro di Teoria dei Sistemi of the National Research Council (C.N.R.) and that of the Ministry of Education (M.P.I.) is gratefully acknowledged.

Sergio Bittanti

ABSTRACTS

Chapter 1 TIME SERIES AND STOCHASTIC MODELS

by E.J. Hannan

The basic concept of this paper is a linear system wherein an output $y(t)$, of q components, is related to an input, $u(t)$, of p components via a relation

$$y(t) = \sum_0^{\infty} W_i e(t-i) + \sum_1^{\infty} L_i u(t-i)$$

wherein the $e(t)$ are the linear prediction errors for $y(t) - \sum_1^{\infty} L_i u(t-i)$. The methods of the paper are substantially valid when the system is truly linear in the sense that linear prediction is optimal, but may prove useful over a much wider range.

To bring the problem back to reasonable proportions the statistical methods are based on the approximation of the true structure by one wherein the matrix functions

$$W(z) = \sum W_i z^{-i}, \quad L(z) = \sum L_i z^{-i}$$

are approximated by matrices of rational functions. A brief discussion is given of some basic theory relating to such an approximation process. It is necessary, in the approximation to choose the "order" of the approximant, i.e. effectively the maximum lags in the ARMAX model,

$$\sum_0^h A_i y(t-i) = \sum_1^h B_i u(t-i) + \sum_0^h C_i e(t-i),$$

to which the rational transfer function corresponds. Various algorithms are described that are basic in time series analysis and are then used to effect a solution to the problem of finding a suitable approximant. The main algorithm described does this by a Gauss-Newton iteration in which the order is redetermined at each iteration by a calculation recursive in the order. Finally, on-line implementations of the algorithm are presented for the case where $y(t)$ is scalar.

Chapter 2 LINEAR ERRORS-IN-VARIABLES SYSTEMS

by M. Deistler

Linear errors-in-variables (EV) systems, i.e. linear systems where all observed variables are subject to errors are considered. The statistical analysis of such systems turns out to be significantly more complicated compared to conventional errors in equations (e.g. ARMAX) systems. A good part of these complications arises from the fact that the transfer function of the system in the EV case, in general, is not uniquely determined from the second moments of the observations.

The paper is organized as follows: In section 2 some well known results concerning the static case are restated. In sections 3 - 5 the information about the transfer function contained in the (ensemble) second moments of the observations is analysed: In section 3 the set of all transfer functions

corresponding to given second moments of the observations is described. Section 4 deals with the same problem when the system is a priori known to be causal and with the problem whether causality can be detected from the second moments of the observations. In section 5 conditions for identifiability are derived. Section 6 deals with conditions for identifiability using information coming from moments of order greater than two.

Chapter 3 A NEW CLASS OF DYNAMIC MODELS
FOR STATIONARY TIME SERIES

by G. Picci and S. Pinzoni

A new class of dynamic models for stationary time series is presented. They are a natural generalization of the well-known linear *Factor Analysis* Models widely used in Statistics and Psychometrics. It is shown that the Factor Analysis Models of time series considered in this note reduce to (and to some extent clarify the structure of) *Dynamic Errors-In-Variable Models* discussed in the recent literature. They provide simple mathematical schemes for the identification of multivariate time series which avoid the unjustified introduction of a priori *causality* assumptions as for example subsumed by conventional ARMAX models.

Chapter 4 PREDICTIVE AND NONPREDICTIVE
MINIMUM DESCRIPTION LENGTH PRINCIPLES

by J. Rissanen

This chapter presents in a tutorial manner the basic ideas behind the recently developed estimation principle, called Minimum Description Length principles. Briefly, a statistical model is judged by the number of binary digits with which it permits one to encode the observed data. The shortest code length available for models in a class is defined to be the stochastic complexity of the data. Depending on how the coding is done two kinds of stochastic complexities can be defined, the predictive and the nonpredictive ones, which for large samples tend to the same value. The stochastic complexity also sets a tight lower bound for the errors with which the data can be predicted. The model associated with the complexity involves estimates both of the number of the parameters and their values, which may be taken to incorporate all the statistical information that can be extracted from the data with the considered models. Hence, we may say that the fundamental problems in modeling are to calculate the stochastic complexity and the associated optimal model.

As applications we describe the calculation of the stochastic complexity of the data relative to the gaussian ARMA class of models, both in the single and the multiple input/output case. We illustrate with simulations the consistency of the associated estimates of the number of the parameters and the structures. We also describe how the prior knowledge about the parameters, as represented by their estimated values, can be taken advantage of. The feasibility of the scheme is demonstrated by simulations.

Chapter 5 DETERMINISTIC AND STOCHASTIC
LINEAR PERIODIC SYSTEMS

by S. Bittanti

The main results concerning the structural properties of linear periodic systems are reviewed. Both continuous-time and discrete-time systems are dealt with. By a comparison with time-invariant systems, five structural properties are discussed. Three of them are basic properties concerning the reachability and controllability subspaces. The fourth one concerns the length of the time interval required to perform the reachability and controllability transition. The modal (spectral) characterizations are presented as fifth property. The extended structural properties (i.e. stabilizability and detectability) are also dealt with. Finally, periodic stochastic systems are considered. The existence of a cyclostationary solution is investigated by analyzing the appropriate periodic Lyapunov equation.

Chapter 6 NUMERICAL PROBLEMS IN LINEAR SYSTEM THEORY

by D. Boley and S. Bittanti

We discuss some numerical aspects in linear system theory. We start by showing the numerical algorithm to solve systems of linear equations and non-degenerate least squares problems. We then move on to an introduction to more sophisticated matrix decompositions, used to solve more sophisticated problems, and introduce the concept of *backward error analysis* (Wilkinson, 1965). Among the decompositions we introduce

<u>name</u>	<u>form</u>	<u>used to obtain</u>
LU (Gaussian Elimination)	$A=LU$. solution of linear Equations . determinant
QR (orthogonal triangularization)	$A=QR$. soln. to least Squares problem (linear non degenerate) . soln. to linear Equations without need to pivot
Schur	$A=QRQ'$. Eigenvalues/vectors
Singular Value Decomposition (S.V.D.)	$A=P\Sigma Q'$. Singular Values . rank . distance to singularity . 2-norm of matrix . 2-norm condition number

where P,Q denote orthogonal matrices

U,R " upper triangular matrices

L " lower triangular matrices

Σ is non-negative diagonal

In the last section we discuss some numerical aspects in linear system theory. The attention is focused on the problem of computing the controllable subspace of a time-invariant linear system. It is shown how some classical methods lead to numerical problems and give some recent results giving bounds on the errors in terms of results from these classical methods.

Chapter 7 SOME RECENT DEVELOPMENTS IN ECONOMETRICS

by M. McAleer and M. Deistler

In this paper we discuss some of the main recent developments in econometrics: methods for specification search, in particular, diagnostic checking and specification testing; macroeconomic modelling and forecasting; and some models associated with empirical microeconomics.

AUTHORS

Sergio Bittanti
Dipartimento di Elettronica
Politecnico di Milano
Piazza Leonardo da Vinci, 32
20133 MILANO
ITALY

Daniel Boley
Department of Computer Science
University of Minnesota
136 Lind Hall
207 Church Street S.E.
MINNEAPOLIS, Minnesota 55455
U.S.A.

Manfred Deistler
Institut für Ökonometrie und
Operations Research
Technische Universität Wien
Argentinierstrasse 8/119
A-1040 WIEN
AUSTRIA

Edward G. Hannan
Department of Statistics
Mathematical Sciences Bldg.
The Australian National University
GPO Box 4 CANBERRA, ACT 2601
AUSTRALIA

Michael J. McAleer
Department of Statistics, The Faculties
The Australian National University
GPO Box 4 CANBERRA, ACT 2601
AUSTRALIA

Giorgio Picci
Istituto di Elettrotecnica ed Elettronica
Università di Padova
Via Gradenigo 6/A
35131 PADOVA
ITALY

Stefano Pinzoni
LADSEB-CNR
Corso Stati Uniti 4
35020 PADOVA
ITALY

Jorma Rissanen
IBM-RES
650 Harry Road
SAN JOSE, CA 95193
U.S.A.

TABLE OF CONTENTS

<u>Chapter 1</u>	TIME SERIES AND STOCHASTIC MODELS	1
	by E.J. Hannan	
1.	Introduction	1
2.	Some Basic Algorithms	4
3.	Approximation Criteria	8
4.	Rational Transfer Function Approximation	12
5.	A. Gauss-Newton Procedure	16
6.	Some Theoretical Considerations	28
	References	34
<u>Chapter 2</u>	LINEAR ERRORS-IN-VARIABLES SYSTEMS	37
	by M. Deistler	
1.	Introduction	37
2.	The Static Case	41
3.	Second Moments and Dynamic Models: the General Case	48
4.	Causality	52
5.	Conditions for Identifiability from the Second Moments of the Observations	58
6.	Identifiability from High Order Moments	63
	References	66

<u>Chapter 3</u>	A NEW CLASS OF DYNAMIC MODELS FOR STATIONARY TIME SERIES by G. Picci and S. Pinzoni	69
1.	Introduction	69
2.	Dynamic Factor Analysis Models	80
3.	Stochastic Realization	87
4.	Causality	104
	References	112
<u>Chapter 4</u>	PREDICTIVE AND NONPREDICTIVE MINIMUM DESCRIPTION LENGTH PRINCIPLES by J. Rissanen	115
1.	Introduction	115
2.	Coding and Prediction	120
3.	ARMA Estimation and Prediction	125
4.	Vector Time Series Models	131
	References	137
<u>Chapter 5</u>	DETERMINISTIC AND STOCHASTIC LINEAR PERIODIC SYSTEMS by S. Bittanti	141
1.	Introduction	141
2.	Structural Properties of Continuous-time Periodic Systems	143

2.1	Continuous-time Linear Periodic Systems	143
2.2	Structural Properties	145
2.3	Grammian Matrices	146
2.4	Five Structural Properties of Time-invariant Systems	146
2.5	Five Structural Properties of Continuous-time Periodic Systems	148
3.	Structural Properties of Discrete-time Periodic Systems	156
3.1	Discrete-time Linear Periodic Systems	156
3.2	Structural Properties	158
3.3	Grammian Matrices	158
3.4	Five Structural Properties of Discrete-time Periodic Systems	159
4.	Kalman Canonical Decomposition	163
5.	Extended Structural Properties	165
6.	Stochastic Linear Periodic Systems	168
	References	176
<u>Chapter 6</u>	NUMERICAL PROBLEMS IN LINEAR SYSTEM THEORY	183
	by D. Boley and S. Bittanti	
1.	Introduction	183
2.	Review of Simpler Computational Methods	183
2.1	LU Decomposition	183
2.2	Orthogonal Decomposition	188
2.2.1	QR Decomposition	188
2.2.2	Geometric Interpretation of a Rotation	191
2.2.3	QR Decomposition by Housolder decompositions	192
2.2.4	Solving Least Squares Problems	194
	Using Orthogonal Decompositions	

3. Special Forms Used in Numerical Linear Algebra-Why	196
3.1 The Jordan Canonical Form	196
3.2 Numerical Conditioning of a Problem	197
4. Schur Decomposition	199
5. Singular Value Decomposition - Condition Number of a Matrix	201
6. Applications of Previous to Linear Systems	211
References	220

Chapter 7 SOME RECENT DEVELOPMENTS IN ECONOMETRICS 222
by M. McAleer and M. Deistler

1. Introduction	222
2. Specification and Quality Control of a Model	226
2.1 Model Specification	227
2.2 Tight and Loose Specifications	228
2.3 Principles for Testing	231
2.4 Diagnostic Testing	232
2.4.1 Serial Correlation	232
2.4.2 Heteroscedasticity	233
2.4.3 Exogeneity	234
2.4.4 Functional Form	234
2.4.5 Parameter Constancy	235
2.4.6 Non-nested Alternatives	235
3. Macroeconomic Modelling and Forecasting	236
4. Microeconometrics	240
References	241